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Wall to suspension heat transfer in the similar profile regime

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Abstract—Probabilistic multiphase flow equations are used to analyse heat transfer between a pipe wall and a gas–solids suspension flowing inside. This analysis shows that, in the same way as self-similar concentration and velocity profiles are solutions to the hydrodynamic equations for dilute suspensions, self-similar temperature profiles constitute solutions to the energy equations. These solutions, in turn, allow the derivation of the general form of the variation in suspension heat transfer coefficient as a function of solids loading in gas–solids suspension flows. The predicted variation is consistent with experimental data available in the literature.

1. INTRODUCTION

INTEREST in the heat transfer behaviour of flowing gas–solids suspensions originated in the 1950s when graphite suspensions were being considered for use as cooling media in nuclear reactors. The early work done in this area consisted of developing correlations between the suspension-to-wall heat transfer coefficients and suspension properties. As applications grew to include transport reactors and circulating fluid beds, experimental studies continued and early attempts at providing a theoretical foundation for the understanding of gas–solids heat transfer were advanced. One of the difficulties encountered in developing both generalized correlations and consistent theoretical approaches to this problem was a curious observation of how heat transfer coefficients varied with solids loading ratio: in some cases, coefficients increased monotonically with loading ratio; in others, coefficients decreased initially as solids were added to the flowing gas stream and then increased after passing through a minimum; in still other cases, the decrease with loading ratio was monotonic.

The probabilistic multiphase flow equations [1] were developed in order to provide a rigorous theoretical framework for the understanding of gas–solids suspension hydrodynamics and heat transfer. In an earlier paper [2], these equations were applied to the hydrodynamics of dilute, fully developed, vertically flowing suspensions. This analysis led to the identification of a 'similar profiles' flow regime where solids velocity and concentration profiles are self-similar. This, in turn, explained certain characteristics of the pressure drop behaviour of vertically flowing suspensions. In this paper, the properties of the temperature fields in the similar profiles regime will be examined and a general heat transfer equation describing the variations of the wall-to-suspension heat transfer coefficient with the loading ratio will be derived.

2. PREVIOUS WORK

The earliest published study on heat transfer between flowing gas–solids suspensions and pipe walls is that of Farbar and Morley [3]. These authors studied 50 μm catalyst particles and found the ratio of suspension heat transfer coefficient over that of the gas alone flowing at the same superficial gas velocity varied with the solids loading ratio (W_s/W_g) to the 0.45 power. Depew and Farbar [4] found that solids had little effect on heat transfer coefficients for 200 μm glass beads below loading ratios of 7. Using 30 μm glass beads however, the same authors observed there to be no effect on heat transfer coefficients up to loading ratios of 0.5. Between 0.5 and 3.0 however, heat transfer coefficients decreased followed by large increase thereafter. This latter observation, whereby coefficients pass through a minimum somewhere in the range $0.5 < W_s/W_g < 3.0$ was later substantiated by numerous workers in a wide variety of situations

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NOMENCLATURE

a, b	compound shape factors defined by equation (50)
C	heat capacity [J kg^{-1} K^{-1}]
f	reduced concentration profile
G	velocity defined in equation (17) [m s^{-1}]
h	heat transfer coefficient [W m^{-2} K^{-1}]
K	axial temperature gradient [K m^{-1}]
O	order of magnitude
Q	rate of heat transfer per unit volume of suspension [W m^{-3}]
r	radial coordinate [m]
R	pipe radius [m]
t	time [s]
T	temperature [K]
U	superficial gas velocity [m s^{-1}]
V	velocity [m s^{-1}]
W	mass flow rate [kg s^{-1}]
x	axial coordinate [m]
Y	defined by equation (51).

Greek symbols

α	phase presence probability
Γ	solids loading times heat capacity ratio (equation (25))
ε	velocity–temperature cofluctuation vector [m K s^{-1}]
ζ	radial heat transfer fluxes defined by equations (27) and (28) [W m^{-2}]
θ	temperature difference [K]
λ	thermal conductivity [W m^{-1} K^{-1}]
Λ	dimensionless shape factors defined by equations (45) and (46)
ρ	phase density [kg m^{-3}]
φ	heat flux [W m^{-2}].

Subscripts

f	fluid phase
i, j	tensor notation
I	interfacial mean
m	mixture
Mm	mixed mean (for suspension temperature)
p	phase p
s	solids phase
w	at the pipe wall.

Superscripts

f	fluid phase
I	interfacial mean
p	phase p
s	solids phase
0	reference flow
1	perturbation flow
$\overline{}$	mean.

(e.g. refs. [5–9]). There are exceptions to be found however in the works of Danziger [10] who found monotonic increases as a function of loading ratio for cracking catalyst suspensions. Kim and Seader [11] found monotonic decreases in coefficients as a function of loading ratio at high Reynolds numbers; they found constant coefficients at Reynolds numbers below 23 000.

Many intuitive explanations have been put forth for this apparently inconsistent behaviour: particles are said to thin the viscous sub-layer in cases where coefficients increase; particles are said to dampen turbulent diffusive exchanges in cases where coefficients fall. One of the few attempts to treat this problem theoretically for isothermal pipe walls was by Tien [12] whose analysis succeeded in predicting that solids have the effect of increasing thermal entry lengths and therefore increasing overall coefficients in short pipes. Tien's equations were later used by Depew and Farbar [4] for uniform wall heat fluxes. They managed to predict the presence of a minimum at low solids loading but predicted no change compared to the coefficient of the gas alone at higher loadings. Use of Tien's equations for fully developed flow required making a large number of simplifying assumptions, including uniform solids concentration profiles and identical solids and gas velocities. The analysis by Matsumoto *et al.* [13] differs from the preceding by the attempt to take into account eddy diffusivity. A recent alternative approach by Michaelides [14] considers the suspension as a variable density, variable heat capacity one-phase fluid. None of these attempts have begun with rigorously derived multiphase flow equations and all have been limited by simplifying assumptions hampering their ability to predict the general behaviour of heat transfer coefficients in thermally fully developed gas–solids suspension flows.

3. THERMALLY FULLY DEVELOPED FLOW OF A VERTICAL GAS–SOLIDS SUSPENSION

3.1. *General equations*

In an earlier work [1], a probabilistic Eulerian description of multiphase flow was developed in order to provide a rigorous mathematical description of multiphase mixtures. According to this approach, the presence and all physical quantities (i.e. immediate Eulerian variables) of each phase are assumed to be random variables governed by laws of probability determined by the overall boundary conditions imposed on the flow. A given phase p in the mixture is then characterized by a probability of presence α_p and probabilistic mean Eulerian variables each of them being defined as the expected value of the random immediate Eulerian variable under consideration. The general probabilistic multiphase flow equations are written in terms of these 'phase mean variables' which are identifiable to measurable quantities in the flow.

When all the particles in the suspension have the same physical properties (i.e. density ρ_s, heat capacity C_s, thermal conductivity λ_s) regardless of their size or shape, they can be regrouped in a unique solids phase. The suspension can then be considered to be a two phase mixture, one solids, one fluid. Each of the two phases of the mixture has its specific continuity, momentum and energy equations. The latter simplifies slightly when the temperature levels in the system are low enough so that radiation can be neglected compared to convective heat transfer. Moreover, when only moderate temperature differences exist in the flow, the physical properties of the phases can be considered as essentially constant. Finally, the work done by the internal forces is generally negligible compared to the power transferred to the suspension. The energy equation of a phase p then simplifies to

$$\frac{\partial}{\partial t}(\rho_p\alpha_pC_pT_p)+\frac{\partial}{\partial x_i}[\rho_p\alpha_pC_p(V_i^pT_p+\varepsilon_i^p)] = \frac{\partial}{\partial x_i}\left[\lambda_p\frac{\partial}{\partial x_i}(\alpha_pT_p)-\lambda_pT_p^I\frac{\partial\alpha_p}{\partial x_i}\right]+Q_p. \quad (1)$$

The divergence term in the LHS of equation (1) accounts for the heat transported by the phase p. It includes convective transport in terms of phase mean variables (first term within parenthesis) and the heat transported by the random movement of the phase characterized by the velocity–temperature cofluctuation vector ε_i. The first term on the RHS (which is also a divergence) represents the heat transferred by conduction within the phase p; its complete expression includes a term (second term within brackets) depending on the 'interfacial mean temperature' of the phase. The latter is determined by the interfacial jump conditions. Finally the last term Q_p in equation (1) is the heat transferred to the interfacial surface of the phase p per unit volume of suspension.

For a gas–solids two-phase mixture with negligible radiation, the following holds:

$$Q_f = -Q_s = -Q \quad (2)$$

since the heat received by the particles comes entirely from the fluid. This jump condition results from the fact that the particles cannot receive heat from the walls or from other particles except by radiation, provided that the contacts of particle interfaces with rigid surfaces are necessarily point contacts which would require an infinite temperature gradient to give a nonzero term in the phase mean equations [15]. Another jump condition is the continuity of the temperatures at a gas–particle interface. In terms of phase mean variables it results in the following equation expressing the identity of the 'interfacial mean' temperatures:

$$T_f^I = T_s^I = T_I. \quad (3)$$

An additional equation relates the probabilities of presence of both phases:

$$\alpha_f+\alpha_s = 1. \quad (4)$$

These phase presence probabilities have been shown to be equivalent to the local volume fractions occupied by the phases [15].

There is an obvious closure problem related with the system made of the energy equations of both phases together with the coupling equations (2)–(4).

3.2. *Thermally fully developed flow*

Consider a suspension flowing upward with constant gas and solids flow rates in a rectilinear vertical pipe of circular cross-section. If the thermal boundary conditions are axisymmetric, the concentration, velocity and temperature fields of both phases will be axisymmetric. Moreover, beyond a certain distance downstream from the entrance the flow patterns are identical in each cross-section of the pipe (i.e. independent of the axial coordinate x). The latter situation is called *fully developed flow* and has been characterized in a previous paper [2]. Molodtsof [15] proposed the following definition for the *thermally* fully developed flow of a gas–solids suspension:

(i) the flow is steady and dynamically fully developed;

(ii) the shape of the transverse temperature profile of each phase is independent of x; and

(iii) for each phase, the probability distribution laws of the random temperatures about the phase mean temperature are independent of x.

According to the first condition, the flow is one directional and the phase velocities as well as the phase presence probabilities are functions of the radial coordinate r only [2]. From the second condition, the following equations can be derived for the fluid and solids phases, respectively:

$$\theta_f(r) = T_w(x)-T_f(x,r) \quad (5)$$

$$\theta_s(r) = T_w(x)-T_s(x,r) \quad (6)$$

where $T_w(x)$ is the wall temperature and the functions θ_f and θ_s represent the transverse temperature profiles. A similar equation can be written for the 'interfacial mean temperature' according to the third condition

$$\theta_I(r) = T_w(x)-T_I(x,r). \quad (7)$$

Finally the interphase heat transfer term Q is also a function of r only as it follows from the third condition above.

The energy equations derived from equation (1), respectively, for the fluid and solids thus take the following simplified form:

$$\rho_fC_f\left[\alpha_fV_f\frac{dT_w}{dx}+\frac{1}{r}\frac{d}{dr}(r\alpha_f\varepsilon_r^f)\right] = -\lambda_f\left[\frac{1}{r}\frac{d}{dr}(r\alpha_f\theta_f)-\theta_I\frac{d\alpha_f}{dr}\right]-Q \quad (8)$$

and

$$\rho_s C_s\left[\alpha_s V_s \frac{dT_w}{dx} + \frac{1}{r}\frac{d}{dr}(r\alpha_s \varepsilon_r^s)\right] = -\lambda_s\left[\frac{1}{r}\frac{d}{dr}(r\alpha_s\theta_s) - \theta_1\frac{d\alpha_s}{dr}\right] + Q \quad (9)$$

where V_f and V_s denote the non-zero (namely, axial) components of fluid and solids velocities, respectively. In these equations, all the terms are independent of x. The axial gradient of the wall temperature should, therefore, be a constant. The physical significance of this necessary condition will be seen below in the heat balance.

3.3. *Heat balance and interphase transfer*

An overall heat balance equation can be derived by adding equations (8) and (9), and integrating the sum over the cross-section; using the following boundary conditions at the wall [15]:

$$\alpha_s = 0 \quad \text{for } r = R \quad (10)$$

$$\frac{d\alpha_s}{dr} = 0 \quad \text{for } r = R \quad (11)$$

$$\theta_f = 0 \quad \text{for } r = R \quad (12)$$

$$\varepsilon_r^f = 0 \quad \text{for } r = R \quad (13)$$

where R denotes the pipe radius. Equations (10) and (11) signify that the wall is impenetrable and particle/wall contact is limited to discrete points.

The integration leads to:

$$(W_f C_f + W_s C_s)\frac{dT_w}{dx} = 2\pi R \varphi_w \quad (14)$$

where, W_f and W_s denote, respectively, the fluid and solids mass flowrates and φ_w represents the unit heat flux provided by the wall to the suspension:

$$\varphi_w = -\lambda_f \frac{d\theta_f}{dr} \quad \text{for } r = R. \quad (15)$$

Equation (14) shows that meeting the necessary condition for thermally fully developed flow, namely, that there be a constant axial temperature gradient at the wall, can only be accomplished if *a uniform heat flux is provided between the suspension and the wall.*

According to equation (14), φ_w is a boundary condition which determines the axial temperature gradient for given hydrodynamic operating conditions. When the flow field is known, the radial temperature fields θ_f and θ_s can be determined by solving equations (8) and (9) with boundary conditions given by equations (10)–(13) and φ_w, provided that the velocity–temperature cofluctuation vectors, the interfacial mean temperature and the interphase heat transfer term Q are expressed in terms of the basic variables (i.e. the phase mean velocity and temperatures). Such closure equations could be deduced from a detailed modelling of the probability distributions of random velocities and temperatures. At the present stage of our knowledge, however, these probability distributions are unknown. Nevertheless, an expression for Q can be derived from the following argument. When equation (9) alone is integrated over a cross-section, the relationship between the cross-sectional average of Q and the axial temperature gradient is

$$\bar{Q} = \frac{1}{\pi R^2} W_s C_s \frac{dT_w}{dx}. \quad (16)$$

This suggests that Q is governed by the same axial temperature scale as the phase mean enthalpies. It cannot reasonably be assumed, however, without considerable loss of generality, that the local value $Q(r)$ is at any point equal to the average given by equation (16). Previous authors assumed that Q is proportional to the local solids volumetric capacity flux $(\rho_s\alpha_s C_s V_s)$ but this implies, as can be seen by equation (9), that there is no radial heat transfer due to the particles. The interphase heat transfer Q acts, indeed, as a heat sink in equation (8). Since the heat transferred by the gas to the particles has to be stored in the solids phase prior to radial transfer by conduction within the phase and/or random transport in the radial direction, it seems reasonable to consider $Q(r)$ as being proportional to the local volumetric heat capacity of the particles $(\rho_s\alpha_s C_s)$. But the additional factor $G(r)$ should be different from $V_s(r)$ in order to allow a radial heat flux due to the particles:

$$Q(r) = \rho_s\alpha_s C_s G(r)\frac{dT_w}{dx} \quad (17)$$

$G(r)$ has the dimensions of a velocity but differs from $V_s(r)$.

4. TEMPERATURE FIELDS IN SIMILAR PROFILE REGIME

4.1. *Similar profile regime*

In a previous paper [2] it was shown that the dilute phase fully developed flow of a vertical gas–solids suspension is characterized by self-similar solids concentration and velocity profiles

$$\alpha_s(r) = \bar{\alpha}_s f(r; U) \quad (18)$$

$$V_s(r) = V_s(r; U) \quad (19)$$

at a constant superficial gas velocity U. In fact, the profiles in equations (18) and (19) are *asymptotic solutions to the general equations when the average solids volumetric concentration tends toward zero.* The latter is defined by:

$$\bar{\alpha}_s = \frac{1}{\pi R^2}\int_0^R 2\pi r\alpha_s(r)\,dr. \quad (20)$$

Experimental results cited in ref. [2] show that solids velocity and 'reduced' concentration profiles (i.e. concentration profiles divided by the mean concentration) which are independent of solids loading have been reported by many authors for average volumetric particle concentrations up to a few per cent. The *similar profile regime* seems, therefore, to be a typical feature of 'dilute' phase flow. In this regime, the gas velocity profiles deform with increasing solids concentration according to the following asymptotic form:

$$V_{\mathrm{f}}(r) = V_{\mathrm{f}}^0(r; U) + \alpha_{\mathrm{s}}(r; U)V_{\mathrm{f}}^1(r; U) + O(\overline{\alpha_{\mathrm{s}}}^2). \quad (21)$$

In fact, equations (18), (19) and (21) are MacLaurin series expansions limited to the first order in terms of increasing powers of the average solids concentration. In general, the difference between the reference flow variables (e.g. V^0) and the suspension flow variables (e.g. V) are as a result of modifications due to the presence of the particles.

4.2. *Temperature field of the reference flow*

In equation (21), the first term in the RHS is the velocity field of the *reference flow* (i.e. the velocity profile of the gas flowing alone in the same pipe with the same superficial velocity). The basic hypothesis used in the previous hydrodynamic analysis [2] is that all Eulerian variables of the fluid tend toward their values in the reference flow when the average solids concentration tends toward zero. For the temperature fields, the same should hold for both the axial and radial profiles. The energy equation for the reference flow is

$$\rho_{\mathrm{f}}C_{\mathrm{f}}\left[V_{\mathrm{f}}^0\frac{\mathrm{d}T_{\mathrm{w}}^0}{\mathrm{d}x} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\varepsilon_{\mathrm{r}}^{\mathrm{f}0})\right] = -\lambda_{\mathrm{f}}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\theta_{\mathrm{f}}^0}{\mathrm{d}r}\right) \quad (22)$$

where the '^{0}' superscript designates the reference flow variables. The temperature fields are defined by the wall heat flux. Consequently, the temperature field of the reference flow should be that obtained with the same thermal boundary conditions as those imposed on the suspension: the same inlet temperature and the same wall heat flux φ_{w}. The limiting form of equation (15) is, therefore

$$\varphi_{\mathrm{w}} = -\lambda_{\mathrm{f}}\frac{\mathrm{d}\theta_{\mathrm{f}}^0}{\mathrm{d}r} \quad \text{for } r = R. \quad (23)$$

The heat balance equation can be obtained by letting $W_{\mathrm{s}} = 0$ in equation (14)

$$W_{\mathrm{f}}C_{\mathrm{f}}\frac{\mathrm{d}T_{\mathrm{w}}^0}{\mathrm{d}x} = 2\pi R\varphi_{\mathrm{w}}. \quad (24)$$

The axial temperature gradient of the reference flow differs, then, from that of the suspension by a scaling factor which can be expressed using the following ratio:

$$\Gamma = \frac{W_{\mathrm{s}}C_{\mathrm{s}}}{W_{\mathrm{f}}C_{\mathrm{f}}}. \quad (25)$$

Now, if K and K^0 denote, respectively, the axial temperature gradients for the suspension and the reference flow, considering equations (14), (24) and (25), one has

$$K = \frac{K^0}{1+\Gamma}. \quad (26)$$

In similar profiles regime Γ tends towards zero as the average concentration tends towards zero. Normally, in gas–solids suspensions the order of magnitude of $\Gamma/\overline{\alpha_{\mathrm{s}}}$ is however, about 10^3.

4.3. *Asymptotic form of the energy equation*

In order to give to equations (8) and (9) a simpler form the following variables will be defined to regroup all the radial heat transfer terms for the fluid and the solids, respectively:

$$\zeta_{\mathrm{f}} = -\lambda_{\mathrm{f}}\left[\frac{\mathrm{d}}{\mathrm{d}r}(\alpha_{\mathrm{f}}\theta_{\mathrm{f}}) - \theta_1\frac{\mathrm{d}\alpha_{\mathrm{f}}}{\mathrm{d}r}\right] - \rho_{\mathrm{f}}\alpha_{\mathrm{f}}C_{\mathrm{f}}\varepsilon_{\mathrm{r}}^{\mathrm{f}} \quad (27)$$

$$\zeta_{\mathrm{s}} = -\lambda_{\mathrm{s}}\left[\frac{\mathrm{d}}{\mathrm{d}r}(\alpha_{\mathrm{s}}\theta_{\mathrm{s}}) - \theta_1\frac{\mathrm{d}\alpha_{\mathrm{s}}}{\mathrm{d}r}\right] - \rho_{\mathrm{s}}\alpha_{\mathrm{s}}C_{\mathrm{s}}\varepsilon_{\mathrm{r}}^{\mathrm{s}}. \quad (28)$$

Energy equations (8) and (9) can be rewritten using these new variables and substituting the expression given by equation (17) for Q. If the equations then obtained are in turn divided by K, one obtains

$$\rho_{\mathrm{f}}C_{\mathrm{f}}\alpha_{\mathrm{f}}V_{\mathrm{f}} = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\zeta_{\mathrm{f}}}{K}\right) - \rho_{\mathrm{s}}C_{\mathrm{s}}\alpha_{\mathrm{s}}G \quad (29)$$

$$\rho_{\mathrm{s}}C_{\mathrm{s}}\alpha_{\mathrm{s}}V_{\mathrm{s}} = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\zeta_{\mathrm{s}}}{K}\right) + \rho_{\mathrm{s}}C_{\mathrm{s}}\alpha_{\mathrm{s}}G. \quad (30)$$

An analogous equation can be derived for the reference flow

$$\rho_{\mathrm{f}}C_{\mathrm{f}}V_{\mathrm{f}}^0 = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\zeta_{\mathrm{f}}^0}{K^0}\right) \quad (31)$$

which is the limiting form of equation (29) as the average concentration tends toward zero.

Consider now the MacLaurin series expansion of G limited to zeroth order terms

$$G(r, \alpha_{\mathrm{s}}, U) = G^0(r; U) + O(\alpha_{\mathrm{s}}). \quad (32)$$

If this expression is substituted for G in equation (30) and second order terms are neglected, dividing through by $\overline{\alpha_{\mathrm{s}}}$ leads to an equation in which the LHS and the second term on the RHS are independent of the average solids concentration (this follows from equations (18) and (19)). Therefore, the asymptotic form of the compound $\zeta_{\mathrm{s}}/(K\overline{\alpha_{\mathrm{s}}})$ is independent of $\overline{\alpha_{\mathrm{s}}}$. Now, if equation (28) is considered, it appears that the solids phase energy equation allows as solutions, asymptotic temperature profiles of the form

$$\alpha_{\mathrm{s}}\theta_{\mathrm{s}} = \frac{\alpha_{\mathrm{s}}\theta_{\mathrm{s}}^1(r; U)}{(1+\Gamma)} + O(\overline{\alpha_{\mathrm{s}}}^2) \quad (33)$$

$$\theta_1 = \frac{\theta_1^0(r; U)}{(1+\Gamma)} + O(\overline{\alpha_{\mathrm{s}}}). \quad (34)$$

It should be noted that, the first term on RHS of equation (33) is not exactly the first order term of a MacLaurin series expansion since Γ depends on $\overline{\alpha_{\mathrm{s}}}$.

But as the ratio of these two latter variables has an order of magnitude much greater than 1 the resulting second order term cannot be neglected.

An analogous result can be derived for the gas temperature field from equations (29) and (31). Subtracting the second from the first one gives

$$\rho_f C_f(\alpha_f V_f - V_f^0) = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[r\left(\frac{\zeta_f}{K} - \frac{\zeta_f^0}{K^0}\right)\right] - \rho_s C_s \alpha_s G. \tag{35}$$

The asymptotic form of the LHS of the above equation can be substituted from equation (21). Again, neglecting the second order terms and dividing through the resulting equation by $\overline{\alpha_s}$ gives

$$\rho_f C_f f(r) V_f^1 = \frac{1}{\alpha_s}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[r\left(\frac{\zeta_f}{K} - \frac{\zeta_f^0}{K^0}\right)\right] - \rho_s C_s f(r) G. \tag{36}$$

As the first and last terms in equation (36) are independent of the average concentration, the derivative is also independent of $\overline{\alpha_s}$. Consequently, the following asymptotic form can be deduced:

$$\frac{\zeta_f}{K} = \frac{\zeta_f^0}{K^0} + \alpha_s \frac{\zeta_f^1}{K^0} + O(\overline{\alpha_s}^2). \tag{37}$$

(The term K^0 has been introduced on the RHS of equation (37) for dimensional considerations.) According to the definition of the radial flux term (equation (27)), the temperature and cofluctuation terms for the fluid should be of the following form:

$$\alpha_f \theta_f = \theta_f^0 + \frac{\alpha_s \theta_f^1}{1+\Gamma} + O(\alpha_s^2) \tag{38}$$

$$\alpha_f \varepsilon_r^f = \varepsilon_r^{f0} + \frac{\alpha_s \varepsilon_r^{f1}}{1+\Gamma} + O(\alpha_s^2). \tag{39}$$

Consequently, equations (35), (38) and (39) satisfy equation (37) and are thus, possible solutions of the energy equation of the gas. Formally, these asymptotic profiles are slightly different from those of the flow patterns defined by equations (18), (19) and (21) by the presence of an additional scaling factor $(1+\Gamma)$ which accounts for the increase of the volumetric heat capacity of the suspension with increasing loading. The Eulerian variables of the solids phase are, however, still self-similar.

5. CONSEQUENCES FOR THE HEAT TRANSFER COEFFICIENT

As shown above, the temperature profiles defined by equations (33), (34) and (38) are *possible* asymptotic solutions for the energy equations in the similar profiles regime. The derivation of these results gives no additional information about the functions making up these expressions. It should be recalled, in this connection, that, even the temperature field of the reference flow is 'unknown' from a theoretical point of view. Nevertheless, the similarity properties of these asymptotic profiles can be used to derive an equation predicting the variations of the wall-to-suspension heat transfer coefficient with solids loading.

5.1. *Reference temperature differences*

The heat transfer coefficient expresses the ratio of the wall heat flux to a reference temperature difference ΔT. For the latter several different definitions (reviewed in ref. [16]) have been used by previous authors. When considering transfer from the wall to the suspension (i.e. the mixture), the most useful definition of ΔT would be the difference between the local wall temperature T_w and the suspension mixed mean temperature, the latter being proportional to the mixture's enthalpy. We shall, therefore, define ΔT_m as follows:

$$\Delta T_m = T_w(x) - T_{Mm}(x). \tag{40}$$

This temperature difference is independent of x and can be computed using equations (5) and (6) and the conventional definition of the mixed mean temperature:

$$(W_f C_f + W_s C_s)\Delta T_m = \frac{1}{\pi R^2}\int_0^R 2\pi r[\rho_f C_f \alpha_f V_f \theta_f + \rho_s C_s \alpha_s V_s \theta_s]\,\mathrm{d}r. \tag{41}$$

Alternative definitions for ΔT can be obtained using, respectively, the mixed mean temperatures of the fluid or the solids phases: ΔT_f and ΔT_s. These three reference temperature differences are related by the following equation:

$$W_f C_f \Delta T_f + W_s C_s \Delta T_s = (W_f C_f + W_s C_s)\Delta T_m. \tag{42}$$

These temperature differences can be expressed, using asymptotic forms of the concentration, velocity and temperature fields defined in equations (18), (19), (21), (33) and (38). Neglecting the second order term gives

$$\Delta T_f = \Delta T^0\left[1 + \alpha_s \Lambda_f^0 + \left(\frac{\alpha_s \Lambda_f^1}{1+\Gamma}\right)\right] \tag{43}$$

$$\Delta T_s = \Delta T^0\left(\frac{\Lambda_s}{1+\Gamma}\right). \tag{44}$$

All the ΔTs are, thus, proportional to the reference temperature difference ΔT^0 of the reference flow. (ΔT^0 is the limiting form of ΔT_f when $\overline{\alpha_s}$ approaches zero.) *The proportionality factors depend exclusively on the hydrodynamic operating conditions*, since the terms denoted by Λs in equations (43) and (44) are dimensionless shape factors accounting for the effect of the non-uniformity of the profiles

$$\Lambda_f^0 = \frac{1}{A}\int_A f(r)\frac{V_f^0 + V_f^1}{U}\frac{\theta_f^0}{\Delta T^0}\,\mathrm{d}A \tag{45}$$

$$\Lambda_f^1 = \frac{1}{A}\int_A f(r)\frac{V_f^0}{U}\frac{\theta_f^1}{\Delta T^0}\,\mathrm{d}A \tag{46}$$

$$\Lambda_s = \frac{1}{A}\int_A f(r)\frac{V_s}{\bar{V}_s}\frac{\theta_s^0}{\Delta T^0}\,\mathrm{d}A. \tag{47}$$

5.2. *Asymptotic equation for the heat transfer coefficient*

With the help of equations (41)–(47) an asymptotic equation expressing the variations of the heat transfer coefficient with the solids loading can be derived. The wall-to-suspension heat transfer coefficient h_m is defined as follows:

$$h_m = \frac{\varphi_w}{\Delta T_m}. \tag{48}$$

It can be expressed in terms of the heat transfer coefficient h_0 of the reference (one phase) flow which is defined in an analogous manner

$$h_0 = \frac{\varphi_w}{\Delta T^0}. \tag{49}$$

This equation can be written as follows:

$$\frac{h_m}{h_0} = \frac{(1+\Gamma)^2}{(1+a\Gamma+b\Gamma^2)} \tag{50}$$

where the coefficients a and b are arithmetic combinations of Λ_f^0, Λ_f^1 and Λ_s, and as such, are compound shape factors determined by the hydrodynamic characteristics of the flow.

6. DISCUSSION

Equation (50) is a general result describing the variation of the suspension heat transfer coefficient as a function of solids loading at constant superficial gas velocity. The primary hypotheses leading to its derivation are:

(i) The existence of a flow regime where solids velocity and concentration profiles, while non-uniform through the pipe cross-section, are self-similar as solids loadings increase. There is significant direct evidence of this at low solids concentrations in the literature from local measurements and indirect evidence exists through pressure drop measurements as discussed in ref. [2].

(ii) The axial heat flux is constant. If a different set of boundary conditions are imposed on the flow, the flow will not be thermally fully developed as indicated in equation (14).

(iii) The existence of self-similar solids temperature profiles, that is profiles whose form is arbitrary but which remain the same as solids loading increases. There is little direct evidence either to support or refute this in the literature but this hypothesis can be indirectly confirmed by experimentally verifying equation (50).

Equation (50) allows the entire impact of the complex hydrodynamics of the flow on the heat transfer behaviour to be summarized by two parameters a and b. These parameters depend only on the reduced velocity, concentration and temperature profiles. The profiles themselves however may be dependent on a variety of factors characterizing a given system (particle size and type, pipe diameter, etc.) as well as the superficial gas velocity. Without a detailed knowledge of these profiles, a and b cannot be calculated directly and remain to be determined experimentally. In order to facilitate this determination, equation (50) can be re-arranged to give equation (51)

$$Y = \frac{1}{\Gamma}\left[\frac{(1+\Gamma)^2}{(h_m/h_0)} - 1\right] = a + b\Gamma. \tag{51}$$

Plotting the LHS of equation (51) as a function of Γ will allow a and b to be determined from the slope and intercept. Figure 1 shows data from Depew and Farbar [4], and Tien and Quan [8] plotted in this way. At constant gas velocity, the relationship is linear indicating that the flow regime characterized by self-similar profiles and, therefore, equation (50) succeeds in predicting the heat transfer behaviour. These data also show that the same values of a and b hold for several different gas velocities. This is not predicted directly from the analysis leading up to equation (50) as, normally, a and b would vary as gas velocity varies. The fact that they are constant in Fig. 1 indicates a relative insensitivity, at least in this case, over a narrow range of gas velocities.

Equation (50) allows for six basic types of variations of heat transfer coefficient as a function of solids loading ratio. This is illustrated in Fig. 2 with the associated ranges of values of a and b plotted in Fig. 3. Three of the types of variations (Types 1, 5 and 6) give coefficients which tend to values less than that of the gas alone at the same superficial gas velocity as solids loading is increased. The other three (Types 2, 3 and 4) lead ultimately to higher values. Two of the variation types (Types 4 and 5) lead to initial decreases in the heat transfer coefficients, passage through a minimum, followed by increases there-

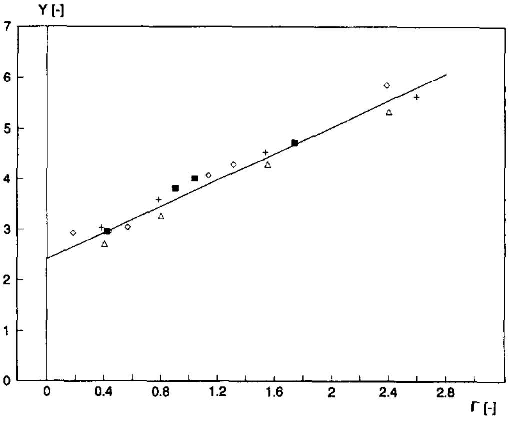

FIG. 1. Comparison between experimental data from Tien and Quan [8] and Depew and Farbar [4] and equation (51). ■ 200 μm glass, Reynolds number = 13 500. + 200 μm glass, Reynolds number = 15 000. ◇ 200 μm glass, Reynolds number = 27 500. △ 200 μm glass, Reynolds number = 30 000.

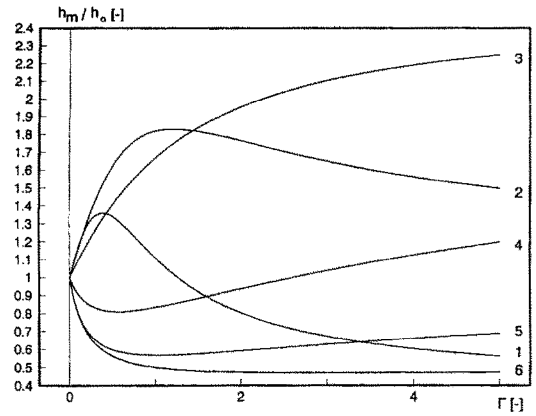

FIG. 2. Possible types of variation of suspension heat transfer coefficient with loading ratio in the similar profile regime depending on the values of *a* and *b*.

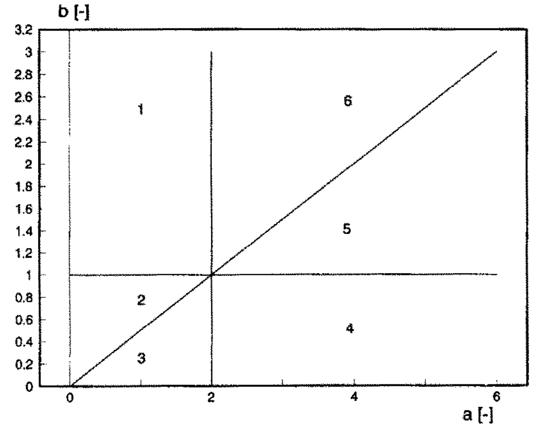

FIG. 3. Grid of possible values of *a* and *b* corresponding to the types of variation shown in Fig. 2.

after. One variation increases monotonically with loading ratio (Type 3); another declines monotonically with increasing loading ratio (Type 6). As indicated earlier, each of these behaviours (with the possible exceptions of Types 1 and 2) have been reported experimentally: in summaries of reported data, both Maeda *et al.* [17] and Boothroyd and Haque [18] show numerous examples of these kinds of behaviour.

7. CONCLUSIONS

Probabilistic multiphase flow equations have been used to analyse heat transfer between the pipe wall and flowing gas–solids suspensions. This analysis has shown that self-similar temperature profiles constitute solutions to the energy equations. These solutions, in turn, allow the derivation of the general form of the variation in heat transfer coefficient as a function of solids loading in gas–solids suspension flow. The general variation, which depends on only two parameters which characterize the detailed hydrodynamics of the flow, is consistent with data available in the literature.

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TRANSFERT THERMIQUE SUSPENSION-PAROI EN RÉGIME DES PROFILS SEMBLABLES

Résumé—Une étude théorique du transfert de chaleur entre une suspension gaz–solides et la paroi du tube qui la vehicule est effectuée á l'aide des Equations Générales Probabilistes des écoulements polyphasiques. On en déduit qu'á l'instar des profils semblables de concentration et de vitesse qui satisfont asymptotiquement aux équations de l'écoulement, les équations de l'énergie admettent comme solution des profils semblables de température, en écoulement dilué. Cette forme de solution permet, ensuite, d'établir l'expression générale de la loi de variation du coefficient d'échange de chaleur avec le taux de charge de la suspension. Cette expression est en excellent accord avec les résultats expérimentaux.

WÄRMEÜBERGANG ZWISCHEN EINER WAND UND EINER SUSPENSION IM BEREICH ÄHNLICHER PROFILE

Zusammenfassung—Wahrscheinlichkeitsgleichungen für die Mehrphasenströmung werden auf die Untersuchung des Wärmeübergangs zwischen einer Rohrwand und einer Gas/Feststoffsuspensionsströmung durch das Rohr angewandt. Die Untersuchung zeigt, daß in der gleichen Weise wie die selbstähnlichen Konzentrations- und Geschwindigkeitsprofile Lösungen für die hydrodynamischen Gleichungen verdünnter Suspensionen sind, auch die selbstähnlichen Temperaturprofile Lösungen der Energiegleichungen darstellen. Diese Lösungen wiederum erlauben es, den Wärmeübergangskoeffizienten für die Suspension in allgemeiner Form als Funktion der Feststoffbeladung der Gas/Feststoffsuspensionsströmung abzuleiten. Die berechnete Abhängigkeit stimmt mit den in der Literatur verfügbaren Versuchsdaten überein.

ТЕПЛОПЕРЕНОС ОТ СТЕНКИ К ВЗВЕСИ В АВТОМОДЕЛЬНОМ РЕЖИМЕ

Аннотация—Вероятностные уравнения многофазных течений используются для анализа теплопереноса между стенкой трубы и течением взвеси газ – твердые тела в трубе. Полученные в результате решения профили концентраций, скоростей и температур являются автомодельными. Эти решения, в свою очередь, позволжют определить теплоперенос как функцию содержания твердых тел в течениях взвесей. Результаты расчетов согласуются с имеющимися в литературе экспериментальными данными.